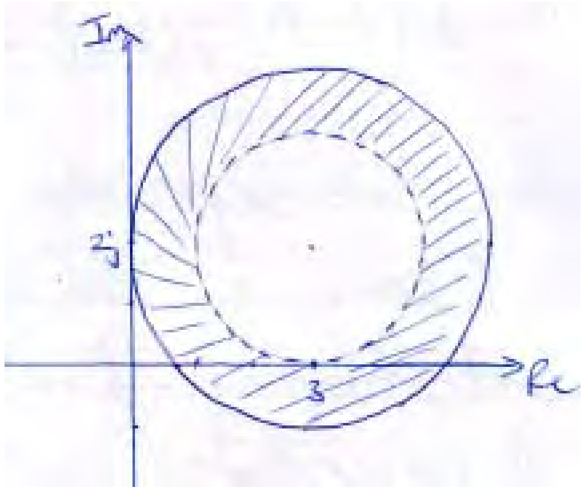
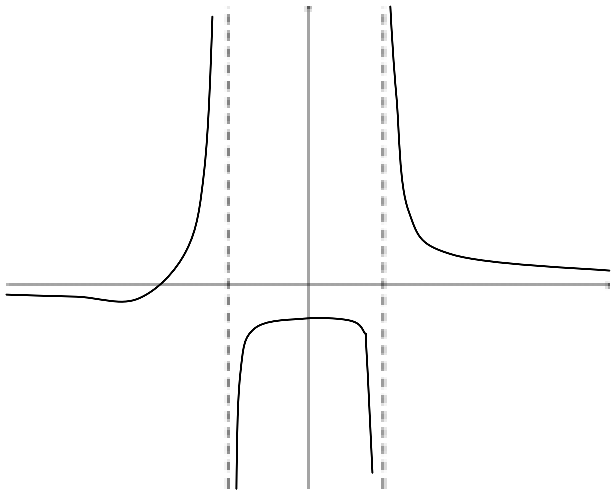
Qu	Answer	Mark	Comment
1	$3x^3+18x^2+Px+31 \equiv Q(x+R)^3+S$		
	$Q=3$	B1	Q = 3 anywhere
	$3x^3+18x^2+Px+31 \equiv 3x^3+9Rx^2+9R^2x+3R^3+S$	M1	Attempt to expand and compare at least another coefficient, or other valid method
	$R=2,\ P=36,\ S=7$	A3	One mark for each correct constant cao
		[5]	
2(i)	$\det \mathbf{M} = 4\times 3-(-1)\times 0$	M1	oe www
	Area $= 12\times 3 = 36$ square units	A1 **[2]**	
2(ii)	$\mathbf{M}^{-1}=\frac{1}{12}\begin{pmatrix}3 & 0\\ 1 & 4\end{pmatrix}$	M1 A1	division by their $\det \mathbf{M}$ cao condone decimals 3sf or better
	$\det \mathbf{M}^{-1}=\frac{1}{12}$	B1 **[3]**	cao condone decimal 3sf or better
2(iii)	$\det \mathbf{M}\times\det \mathbf{M}^{-1}=12\times\frac{1}{12}=1$	B1	Seen or implied
	The inverse 'undoes' the transformation, so the composite of **M** and its inverse must leave a shape unchanged, meaning the area scale factor of the composite transformation must be 1 and so the determinant is 1.	E1	Any valid explanation involving transformations and unchanged area
		[2]	

PMT

3	$\omega = 2x - 1 \Rightarrow x = \dfrac{\omega+1}{2}$ $\left(\dfrac{\omega+1}{2}\right)^3 - 4\left(\dfrac{\omega+1}{2}\right)^2 + 8\left(\dfrac{\omega+1}{2}\right) + 3 = 0$ $\Rightarrow \dfrac{1}{8}\left(\omega^3 + 3\omega^2 + 3\omega + 1\right) - \left(\omega^2 + 2\omega + 1\right)$ $+4\left(\omega+1\right) + 3 = 0$ $\Rightarrow \omega^3 - 5\omega^2 + 19\omega + 49 = 0$	M1 A1 M1 M1 A2 A1 **[7]**	Using a substitution Correct Substitute into cubic Attempting to expand cubic and quadratic LHS oe, -1 each error Correct equation
3	OR $\alpha + \beta + \gamma = 4$ $\alpha\beta + \alpha\gamma + \beta\gamma = 8$ $\alpha\beta\gamma = -3$ Let new roots be *k*, *l*, *m* then $k + l + m = 2\left(\alpha+\beta+\gamma\right) - 3 = 5 = \dfrac{-B}{A}$ $kl + km + lm = 4\left(\alpha\beta + \alpha\gamma + \beta\gamma\right)$ $-4\left(\alpha+\beta+\gamma\right) + 3 = 19 = \dfrac{C}{A}$ $klm = 8\alpha\beta\gamma - 4\left(\alpha\beta + \alpha\gamma + \beta\gamma\right)$ $+2\left(\alpha+\beta+\gamma\right) - 1 = -49 = \dfrac{-D}{A}$ $\Rightarrow \omega^3 - 5\omega^2 + 19\omega + 49 = 0$	M1 A1 M1 A1 A1 A1 A1 **[7]**	Attempt to find $\Sigma\alpha$ $\Sigma\alpha\beta$ $\alpha\beta\gamma$ All correct Attempt to use root relationships to find at least two of Σk Σkl klm Quadratic coefficient Linear coefficient Constant term Correct equation

4		B1 B1 B1 B1 B1 B1 [6]	Circle Centre 3 + 2j Radius = 2 or 3, consistent with their centre Both circles correct cao Correct boundaries indicated, inner excluded, outer included (f t concentric circles) Region between concentric circles indicated as solution SC -1 if axes incorrect
5	$\sum_{r=1}^{n} r^2(3-4r) = 3\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r^3$ $= \frac{3}{6}n(n+1)(2n+1) - \frac{4}{4}n^2(n+1)^2$ $= \frac{1}{2}n(n+1)[(2n+1) - 2n(n+1)]$ $= \frac{1}{2}n(n+1)(1-2n^2)$	M1 M1 A1 M1 A1 [5]	Separate into two sums involving r^2 and r^3, may be implied Appropriate use of at least one standard result Both terms correct Attempt to factorise using both n and $n + 1$ Complete, convincing argument

6	When $n = 1$, $2^{1+1}+1=5$, so true for $n = 1$ Assume $u_k = 2^{k+1}+1$ $\Rightarrow u_{k+1} = 2^{k+1}+1+2^{k+1}$ $= 2\times 2^{k+1}+1$ $= 2^{k+2}+1$ $= 2^{(k+1)+1}+1$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is also true for $k + 1$. Since it is true for $n = 1$, it is true for all positive integers.	B1 E1 M1 A1 E1 E1 **[6]**	 Assuming true for k Using this u_k to find u_{k+1} Correct simplification Dependent on A1 and previous E1 Dependent on B1 and previous E1

7(i)	$\left(0, \frac{-1}{8}\right)$, $(-5, 0)$	B1 B1 **[2]**	One mark for each point SC1 for x = -5, y = -1/8
7(ii)	$x = \frac{5}{2}, x = \frac{-8}{3}, y = 0$	B1 B1 B1 **[3]**	One mark for each equation
7(iii)	Large positive x, $y \to 0^+$ (e.g. consider x = 100) Large negative x, $y \to 0^-$ (e.g. consider x = -100)	B1 B1 M1 **[3]**	Evidence of a valid method
7(iv)		B1 B1 **[2]**	RH branch correct LH branch correct
7(v)	$x < -5$ or $\frac{-8}{3} < x < \frac{5}{2}$	B1 B1 **[2]**	cao cao

8(i)	$\delta = 1 - \text{j}$ There must be a second real root because complex roots occur in conjugate pairs.	B1 E1 **[2]**	
8(ii)	$\alpha + \beta + \gamma + \delta = 1$ $\alpha + \beta + \gamma + \delta = 1 \Rightarrow 1 + (1 + j) + \gamma + (1 - j) = 1$ $\Rightarrow \gamma = -2$	B1 M1 A1 **[3]**	 cao
8(iii)	$(z-1)(z+2)(z-(1+j))(z-(1-j))$ $= (z^2+z-2)(z^2-2z+2)$ $= z^4 - 2z^3 + 2z^2 + z^3 - 2z^2 + 2z - 2z^2 + 4z - 4$ $= z^4 - z^3 - 2z^2 + 6z - 4$ $\Rightarrow a = -2,\ b = 6,\ c = -4$ OR $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = a = -2$ $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -b = -6 \Rightarrow b = 6$ $\alpha\beta\gamma\delta = c = -4$	B1 M1 A3 **[5]** M2 A1 A1 B1 **[5]**	Correct factors from their roots Attempt to expand using all 4 factors One for each of *a*, *b* and *c* Use of root relationships attempted, M2 evidence of all 3, M1 for evidence of 2 OR substitution to get three equations and solving *a* = -2 cao *b* = 6 cao *c* = -4 (SC f t on their 2nd real root)
8(iv)	$\text{f}(-z) = z^4 + z^3 - 2z^2 - 6z - 4$ Roots of $\text{f}(-z) = 0$ are $-1,\ 2,\ -1+\text{j}$ and $-1-\text{j}$	B1 B1 **[2]**	f t on their *a*, *b*, *c*, simplified For all four roots, cao

9(i)	$\mathbf{AB}=\begin{pmatrix}-2 & 1 & -5\\ 3 & a & 1\\ 1 & -1 & 2\end{pmatrix}\begin{pmatrix}2a+1 & 3 & 1+5a\\ -5 & 1 & -13\\ -3-a & -1 & -2a-3\end{pmatrix}$		
	$=\begin{pmatrix}-4a-2-5+15+5a & 0 & 0\\ 0 & 9+a-1 & 0\\ 0 & 0 & 1+5a+13-4a-6\end{pmatrix}$	M1	Attempt to find **AB** with some justification of at least two leading diagonal terms and any other
	$=\begin{pmatrix}8+a & 0 & 0\\ 0 & 8+a & 0\\ 0 & 0 & 8+a\end{pmatrix}$	A1	Correct
	$=(8+a)\mathbf{I}$	B1 **[3]**	Relating correct diagonal matrix to **I**
9(ii)	$\mathbf{A}^{-1}$ does not exist for $a = -8$	B1	
	$\mathbf{A}^{-1}=\dfrac{1}{8+a}\begin{pmatrix}2a+1 & 3 & 1+5a\\ -5 & 1 & -13\\ -3-a & -1 & -2a-3\end{pmatrix}$	M1 A1 **[3]**	k**B,** k not equal to 1 Correct $\mathbf{A}^{-1}$ as shown
9(iii)	$\mathbf{A}^{-1}=\dfrac{1}{12}\begin{pmatrix}9 & 3 & 21\\ -5 & 1 & -13\\ -7 & -1 & -11\end{pmatrix}$	B1	
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\dfrac{1}{12}\begin{pmatrix}9 & 3 & 21\\ -5 & 1 & -13\\ -7 & -1 & -11\end{pmatrix}\begin{pmatrix}-55\\ -9\\ 26\end{pmatrix}=\begin{pmatrix}2\\ -6\\ 9\end{pmatrix}$	M1 A3 **[5]**	Correct use of their $\mathbf{A}^{-1}$ x, y and z cao, -1 each error
9(iv)	There is no unique solution.	B1 **[1]**	